

# Higher order corrections to parton showering from resummation calculations

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## Abstract

The connection between analytic and Monte Carlo calculations of soft gluon emission is reanalyzed in light of recent, theoretical developments in resummation. An alternative Monte Carlo algorithm is suggested which incorporates (1) corrections beyond leading order to the showering and (2) smoothly merges with the higher order calculation of single, hard parton emission. In particular, it is possible to study jet properties in heavy boson production for all values of the boson transverse momentum with a total cross section of NLO accuracy. The specific cases of  $W$ ,  $Z$  and Higgs boson production at the Tevatron are addressed using a modified version of the PYTHIA Monte Carlo.

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## I. INTRODUCTION

In the near future, experiments at two, high-energy hadron colliders will search for evidence of physics that supersedes the standard model. Important among the tools that will be used in these searches are showering event generators or showering Monte Carlos (SMC's). Among the most versatile and popular of these are the Monte Carlos HERWIG [1], ISAJET [2], and PYTHIA [3]. SMC's are useful because they accurately describe the emission of multiple soft gluons, which is, in effect, an all orders problem in QCD. However, they only predict total cross sections to a leading order accuracy, and, thus, can demonstrate a sizeable dependence on the choice of scale used for the parton distribution functions (PDF's) or coupling constants (particularly  $\alpha_s$ ). Also, in general, they do not translate smoothly into kinematic configurations where only one, hard parton is emitted. In distinction to SMC's are certain analytic calculations which account for multiple soft gluon emission and higher order corrections to the hard scattering. These resummation calculations, however, integrate out the kinematics of the soft gluons, and, thus, are limited in their predictive power. They can, for example, describe the kinematics of a heavy gauge boson produced in hadron collision, but cannot predict the number or distribution of jets that accompany it. However, searches for new physics, either directly or indirectly through careful measurements of standard model predictions, often demand detailed knowledge of kinematic distributions and jet activity. Furthermore,  $W$ +jets (and  $Z$ +jets) processes are often backgrounds to SUSY or technicolor signatures, and we demand a reliable prediction of their properties. The aim of this present study is to show how the positive features of the analytic resummation calculations can be used to improve the showering algorithms.

The outline of the remainder of this paper is as follows. First, we review the parton shower (Sec. II) and the analytic resummation formalisms (Sec. III). We then show how to modify the showering algorithm to incorporate higher-order corrections to the total cross section (Sec. IV). Furthermore, we show how to correct the soft gluon approximation made in the showering, so that there is a smooth transition between the showering and hard emission limits (Sec. V), and we compare this approach to other work. Finally, we present numerical results (Sec. VI) for  $W$ ,  $Z$  and Higgs boson production at the Tevatron in Run II using a modified version of PYTHIA, and our conclusions (Sec. VII).

In the ensuing discussion, we focus on the specific example of  $W$  boson production at a hadron collider, when the  $W$  decays leptonically. The results apply equally well to  $\gamma^*$ ,  $Z$  and Higgs bosons (or any heavy, color-singlet particle) produced in hadron collisions.

## II. PARTON SHOWERS

SMC's are based on the factorization theorem [4], which, roughly, states that physical observables in any sensible gauge theory are the product of short-distance functions and long-distance functions. The short-distance functions are calculable in perturbation theory. The long-distance functions are fit at a scale, but their evolution to any other scale is also calculable in perturbation theory.

A standard application of the factorization theorem is to describe  $W$  boson production at a  $p\bar{p}$  collider at a fixed order in  $\alpha_s$ . The production cross section is obtained by convoluting the partonic subprocesses evaluated at the scale  $Q$  with the PDF's evaluated at  $Q$ . The partons involved in the hard collision must be sufficiently virtual to be resolved inside the proton, and a natural choice for the scale  $Q$  is  $Q = M_W$  [5]. Prior to the hard collision, however, the partons are not resolvable in the proton (i.e. the proton is intact) and have virtualities at a much lower scale  $Q_0$  of the order of 1 GeV. The connection between the partons at the low scale  $Q_0$  and those at the high scale  $Q$  is described by the DGLAP evolution equations [6]. The DGLAP equations include the most important kinematic configurations of the splittings  $a \rightarrow bc$ , where  $a, b$  and  $c$  represent different types of partons in the hadron ( $q, g$ , etc.). Starting from a measurement of the PDF's at a low scale  $Q_0$ , a solution of the DGLAP equations yields the PDF's at the hard scale  $Q$ . Equivalently, starting with a parton  $c$  involved in a hard collision, it is also possible to determine probabilistically which splittings generated  $c$ . In the process of de-evolving parton  $c$  back to the valence quarks in the proton, a number of spectator partons (e.g. parton  $b$  in the branching  $a \rightarrow bc$ ) are resolved. These partons constitute a shower of soft jets that accompany the  $W$ -boson, and influence its kinematics.

The shower described above occurs with unit probability and does not change the total cross section for  $W$ -boson production calculated at the scale  $Q$  [7]. The showering can be attached to the hard-scattering process based on a probability distribution *after* the hard scattering has been selected. Once kinematic cuts are applied, the transverse momentum and rapidity of the  $W$ -boson populate regions never accessed by the differential partonic cross section calculated at a fixed order. This is consistent, since the fixed-order calculation was inclusive (i.e.,  $p\bar{p} \rightarrow W + X$ ) and was never intended to describe the detailed kinematics of the  $W$ -boson. The parton shower, in effect, resolves the structure of the inclusive state of partons denoted as  $X$ . In practice, the fixed order partonic cross section (without showering) can still be used to describe properties of the decay leptons as long as the measurable is well defined (e.g., the number of leptons with central rapidity and high transverse momentum,

but not the distribution of rapidity or transverse momentum of the  $W$ ).

Here, we review parton showering schematically. More details can be found in Ref. [8]. First, for simplicity, consider the case of final state or forward showering, where the parton virtuality  $Q$  evolves forward to the low scale  $Q_0$ . The basis for developing a probabilistic picture of final state showering is the DGLAP equation for the fragmentation functions:

$$Q \frac{\partial}{\partial Q} D_a(x, Q) = \int_x^{1-\epsilon} \frac{dz}{z} \frac{\alpha_{abc}(z, Q)}{\pi} \hat{P}_{a \rightarrow bc}(z) D_b(x/z, Q) - D_a(x, Q) \int_x^{1-\epsilon} dz \frac{\alpha_{abc}(z, Q)}{\pi} \hat{P}_{a \rightarrow bc}(z), \quad (2.1)$$

where  $\hat{P}_{a \rightarrow bc}$  is an unregularized splitting function,  $\alpha_{abc}$  is the coupling times color factor, and  $\epsilon$  is a cutoff. The equation can be rewritten as

$$\frac{\partial}{\partial \ln Q^2} (D_a(x, Q)/\Delta(Q)) = \int_x^1 \frac{dz}{z} \frac{\alpha_{abc}(z, Q)}{2\pi} \hat{P}_{a \rightarrow bc}(z) (D_b(x/z, Q)/\Delta(Q))$$

or, after integrating both sides of the expression,

$$D_a(x, t') = D_a(x, t) \Delta(t') + \int_{t'}^t \int_x^1 dt'' \frac{dz}{z} \frac{\Delta(t')}{\Delta(t'')} \frac{\alpha_{abc}(z, t'')}{2\pi} \hat{P}_{a \rightarrow bc}(z) D_b(x/z, t''), \quad (2.2)$$

where  $t = \ln Q^2$ , with similar definitions for  $t'$  and  $t''$ . The function

$$\Delta(t') = \exp \left( - \int_{t_0}^{t'} \int_{\epsilon}^{1-\epsilon} dt'' dz \frac{\alpha_{abc}(z, t'')}{2\pi} \hat{P}_{a \rightarrow bc}(z) \right) \quad (2.3)$$

gives the probability of evolving from the scale  $Q'^2 = e^{t'}$  to  $Q_0^2 = e^{t_0}$  with no resolvable branchings, and is called the Sudakov form factor.  $t_0$  is a cutoff scale for the showering.  $\Delta(t')$  contains all the information necessary to reconstruct a shower, since it encodes the change in virtuality of a parton until a resolvable showering occurs. Showering is reduced to iterative solutions of the equation  $r = \Delta(t')/\Delta(t'')$ , where  $r$  is a random number uniformly distributed in the interval  $[0, 1]$ , until a solution for  $Q'$  is found which is below a cutoff. For consistency, the cutoff should represent the lowest scale of resolvable emission  $Q_0$ .

For the case of initial state radiation, several modifications are necessary. The fragmentation function is replaced by a parton distribution function, and the evolution proceeds backwards from a large, negative scale  $-|Q^2|$  to a small, negative cutoff scale  $-|Q_0^2|$ . There are two equivalent formulations of backwards showering based on the probabilities

$$\exp \left( - \int_{t'}^t \int_{\epsilon}^{1-\epsilon} dt'' dz \frac{\alpha_{abc}(z, t'')}{2\pi} \hat{P}_{a \rightarrow bc}(z) \frac{x' f_a(x', t')}{x f_b(x, t')} \right), x' = x/z \quad [9], \quad (2.4)$$

and

$$\frac{\Delta(t')}{f_b(x, t')} \frac{f_a(x, t'')}{\Delta(t'')} [10]. \quad (2.5)$$

After choosing the change in virtuality, a particular backwards branching is selected from the probability function based on their relative weights (a summation over all possible branchings  $a \rightarrow bc$  is implied these expressions), and the splitting variable is chosen by solving the equation

$$\int_{\epsilon}^{x/x'} \frac{dz}{z} \hat{P}_{a \rightarrow bc}(z) f(x/z, t') = r \int_{\epsilon}^{1-\epsilon} \frac{dz}{z} \hat{P}_{a \rightarrow bc}(z) f(x/z, t'), \quad (2.6)$$

where  $r$  is a random number. The details of how a full shower is reconstructed in the PYTHIA Monte Carlo, for example, can be found in Ref. [3]. The structure of the shower can be complex: the transverse momentum of the  $W$ -boson is built up from the whole series of splittings and boosts, and is known only at the end of the shower, after the final boost.

The SMC formulation outlined above is fairly independent of the hard scattering process considered. Only the initial choice of partons and possibly the high scale differs. Therefore, this formalism can be applied universally to many different scattering problems. In effect, soft gluons are not sensitive to the specifics of the hard scattering, only the color charge of the incoming partons.

### III. ANALYTIC RESUMMATION

At hadron colliders, the partonic cross sections can receive substantial corrections at higher orders in  $\alpha_s$ . This affects not only the total production rate, but also the kinematics of the  $W$  boson. At leading order ( $\alpha_s^0$ ), the  $W$ -boson has a  $\delta(Q_T^2)$  distribution in  $Q_T^2$ . At next-to-leading order, the real emission of a single gluon generates a contribution to  $d\sigma/dQ_T^2$  that behaves as  $Q_T^{-2}\alpha_s(Q_T^2)$  and  $Q_T^{-2}\alpha_s(Q_T^2)\ln(Q^2/Q_T^2)$  while the leading order, soft, and virtual corrections are proportional to  $-\delta(Q_T^2)$ . At higher orders, the most singular terms follow the pattern of  $\alpha_s(Q_T^2)^n \sum_{m=0}^{2n-1} \ln^m(Q^2/Q_T^2) = \alpha_s^n L^m \equiv V^n$ . The logarithms arise from the incomplete cancellation of the virtual and real QCD corrections, but this cancellation becomes complete for the integrated spectrum, where the real gluon can become arbitrarily soft and/or collinear to other partons. The pattern of singular terms suggest that perturbation theory should be performed in powers of  $V^n$  instead of  $\alpha_s^n$ . This reorganization of the perturbative series is called resummation.

The first studies of soft gluon emission resummed the leading logarithms [11,12], leading to a suppression of the cross section at small  $Q_T$ . The suppression underlies the importance of including sub-leading logarithms [13]. The most rigorous approach to the problem of

multiple gluon emission is the Collins–Soper–Sterman (CSS) formalism for transverse momentum resummation [14], which resums all of the important logarithms. This is achieved after a Fourier transformation with respect to  $Q_T$  in the variable  $b$ , so that the series involving the delta function and terms  $V^n$  simplifies to the form of an exponential. Hence, the soft gluon emission is resummed or exponentiated in this  $b$ -space formalism. To be more correct, the Fourier transformation is the result of expressing the transverse-momentum conserving delta functions  $\delta^{(2)}(\vec{Q}_T - \sum \vec{k}_{T_i})$  in their Fourier representation. Also, the exponentiation is accomplished through the application of the renormalization group equation, not by reorganizing an infinite sum of terms. Despite the successes of the  $b$ -space formalism, there are several drawbacks. Most notable for the present study is that it integrates out the soft gluon dynamics and does not have a simple physical interpretation.

The CSS formalism was used by its authors to predict both the total cross section to NLO and the kinematic distributions of the  $W$ -boson to all orders [15] at hadron colliders. A similar treatment was presented using the AEGM formalism [16], that does not involve a Fourier transform, but is evaluated directly in transverse momentum  $Q_T$  space. When evaluated at NLO, the two formalisms are equivalent to NNNL order in  $\alpha_s$ , and agree with the fixed order calculation of the total cross section [17]. A more detailed numerical comparison of the two predictions can be found in Ref. [18].

Recently, the AEGM formalism has been reinvestigated, and an approximation to the  $b$ -space formalism has been developed in  $Q_T$ -space which retains its predictive features [19] (see also the recent eprint [20]). This formulation *does* have a simple, physical interpretation, and can be used to develop an alternate algorithm for parton showering which *includes* higher-order corrections to the hard scattering.

In the  $b$ -space formalism, the differential cross section of the  $W$ -boson produced in association with soft gluons is:

$$\frac{d\sigma(h_1 h_2 \rightarrow V^{(*)} X)}{dQ^2 dQ_T^2 dy} = \frac{1}{(2\pi)^2} \int d^2b e^{i\vec{b} \cdot \vec{Q}_T} \widetilde{W}(b, Q, x_1, x_2) + Y(Q_T, Q, x_1, x_2). \quad (3.1)$$

In this expression,  $Q$ ,  $Q_T$  and  $y$  describe the kinematics of the boson  $V$ , the function  $Y$  is regular as  $Q_T \rightarrow 0$  and corrects for the soft gluon approximation, and the function  $\widetilde{W}$  has the form:

$$\widetilde{W} = e^{-S(b, Q)} \left( C \otimes f \right) (x_1, b) \left( C \otimes f \right) (x_2, b) H(Q, y), \quad (3.2)$$

where

$$S(b, Q, C_1, C_2) = \int_{C_1^2/b^2}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[ \ln \left( \frac{C_2^2 Q^2}{\bar{\mu}^2} \right) A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu})) \right], \quad (3.3)$$

and

$$\left(C_{jl} \otimes f_{l/h_1}\right)(x_1, \mu) = \int_{x_1}^1 \frac{d\xi_1}{\xi_1} C_{jl}\left(\frac{x_1}{\xi_1}, C_1, C_2, \mu = C_3/b\right) f_{l/h_1}(\xi_1, \mu = C_3/b). \quad (3.4)$$

In these expressions,  $C_1$ ,  $C_2$  and  $C_3$  are constants,  $H$  is a function that describes the hard scattering, and  $A$ ,  $B$ , and  $C$  are calculated perturbatively in powers of  $\alpha_s$ :

$$(A, B, C) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu)}{\pi}\right)^n (A, B, C)^{(n)}$$

(the first non-zero terms in the expansion of  $A$  and  $B$  are for  $n = 1$ ). The functions  $C^{(n)}$  are the Wilson coefficients, and are responsible for the change in the total production cross section at higher orders. In fact,  $(C \otimes f)$  is simply a redefinition of the parton distribution function obtained by convoluting the standard ones with an ultraviolet-safe function.

We remove the constants  $C_1$ ,  $C_2$  and  $C_3$  from these expressions by choosing their canonical values [14], which also removes large logarithms. At leading order, the expression for the production of an on-shell  $W$ -boson simplifies considerably to:

$$\frac{d\sigma(h_1 h_2 \rightarrow WX)}{dQ_T^2} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\vec{b} \cdot \vec{Q}_T} e^{-S(b, Q)} \frac{f(x_1, b) f(x_2, b)}{f(x_1, Q) f(x_2, Q)}, \quad (3.5)$$

$$\sigma_0 = \kappa \int \frac{dx_1}{x_1} f(x_1, Q) f(x_2, Q),$$

where  $\kappa$  contains physical constants and we ignore the function  $Y$  for now. The expression contains two factors, the total cross section at leading order  $\sigma_0$ , and a cumulative probability function in  $Q_T^2$  that describes the transverse momentum of the  $W$ -boson (the total integral over  $Q_T^2$  transforms  $e^{i\vec{b} \cdot \vec{Q}_T}$  to  $\delta^{(2)}(\vec{b})$ ). Except for the complication of the Fourier transform, the term  $e^{-S/2} f(x, b)/f(x, Q)$  is analogous to  $\Delta(Q)f(x, Q')/\Delta(Q')f(x, Q)$  of the SMC.

Equation (3.1) in  $b$ -space has a similar structure in  $Q_T$ -space. This is surprising, since the  $b$ -space result depends critically on the conservation of total transverse momentum. To NNNL accuracy, however, the  $Q_T$  space expression agrees exactly with the  $b$ -space expression, and has the form [19]:

$$\frac{d\sigma(h_1 h_2 \rightarrow V^{(*)} X)}{dQ^2 dQ_T^2 dy} = \frac{d}{dQ_T^2} \widetilde{W}(Q_T, Q, x_1, x_2) + Y(Q_T, Q, x_1, x_2). \quad (3.6)$$

Again ignoring  $Y$ , we can rewrite this expression as:

$$\frac{d\sigma(h_1 h_2 \rightarrow WX)}{dQ_T^2} = \sigma_1 \left( \frac{d}{dQ_T^2} \left[ e^{-S(Q_T, Q)} \frac{(C \otimes f)(x_1, Q_T) (C \otimes f)(x_2, Q_T)}{(C \otimes f)(x_1, Q) (C \otimes f)(x_2, Q)} \right] \right), \quad (3.7)$$

$$\sigma_1 = \kappa \int \frac{dx_1}{x_1} (C \otimes f)(x_1, Q) (C \otimes f)(x_2, Q).$$

The factor  $\sigma_1$  is the total cross section to a fixed order, while the rest of the function yields the probability that the  $W$ -boson has a transverse momentum  $Q_T$ .

#### IV. A MODIFIED SHOWERING ALGORITHM

The primary result of this paper is to exploit the expression for the differential cross section in Eq. (3.7), which has the form of a leading order cross section times a backwards evolution. We generalize the function  $\Delta(t)/f(x,t) \times f(x,t')/\Delta(t')$  of the standard backwards showering algorithm to  $\sqrt{\widetilde{W}}$  (the square root appears because we are considering the evolution of each parton line individually).

To implement this modification in a numerical program, like **PYTHIA**, we need to provide the new, modified PDF (mPDF) based on the Wilson coefficients. At leading order, the only Wilson coefficient is  $C_{ij}^{(0)} = \delta_{ij}\delta(1-z)$ , and we reproduce exactly the standard showering formulation. For  $W$ -boson production at NLO, the Wilson coefficients  $C$  are:

$$C_{jk}^{(1)} = \delta_{jk} \left\{ \frac{2}{3}(1-z) + \frac{1}{3}(\pi^2 - 8)\delta(1-z) \right\}, C_{jg}^{(1)} = \frac{1}{2}z(1-z). \quad (4.1)$$

To NLO, the convolution integrals become:

$$(C \otimes f_i)(x, \mu) = f_i(x, \mu) \left( 1 + \frac{\alpha_s(\mu)}{\pi} \frac{1}{3}(\pi^2 - 8) \right) + \frac{\alpha_s(\mu)}{\pi} \int_x^1 \frac{dz}{z} \left[ \frac{2}{3}(1-z)f_i(x/z, \mu) + \frac{1}{2}(1-z)f_g(x/z, \mu) \right], \quad (4.2)$$

and  $f_g(x, \mu)$  is unchanged. The first term gives the contribution of an unevolved parton to the hard scattering, while the other two contain contributions from quarks and gluons with higher momentum fractions that split  $q \rightarrow qg$  and  $g \rightarrow q\bar{q}$ , respectively.

We are relying on the fact that the Sudakov form factor used in the analytic expressions and in the SMC are equivalent. In fact, the integration over the quark splitting function in  $\Delta(Q)$  yields an expression similar to the analytic Sudakov:

$$\int_{z_m}^{1-z_m} dz C_F \left( \frac{1+z^2}{1-z} \right) = C_F \left( \ln \left[ \frac{1-z_m}{z_m} \right]^2 - 3/2(1-2z_m) \right) \simeq A^{(1)} \ln(Q^2/Q_T^2) + B^{(1)}, \quad (4.3)$$

where  $z_m = \frac{Q_T}{(Q+Q_T)}$  is an infrared cutoff, terms of order  $z_m$  and higher are neglected, and the  $z$  dependence of the running coupling has been ignored [21]. Note that the coefficients  $A^{(1)}$  ( $C_F$ ) and  $B^{(1)}$  ( $-3/2C_F$ ) are universal to  $q\bar{q}$  annihilation into a color singlet object, just as the showering Sudakov form factor only knows about the partons and not the details of the hard scattering. For  $gg$  fusion, only the coefficient  $A^{(1)}$  (3) is universal. In general, at higher orders, the analytic Sudakov is sensitive to the exact hard scattering process.

While the Sudakov form factors are similar, there is no one-to-one correspondence. First, the  $Q_T$ -space Sudakov form factor is expressed directly in terms of the  $Q_T$  of the heavy boson, while, in the SMC's, the final  $Q_T$  is built up from a series of branchings.



Secondly, the integral on the left of Eq. (4.3) is positive (provided that  $z_m < \frac{1}{2}$ ), while the analytic expression on the right can become negative. This is disturbing, since it means subleading logarithms (proportional to  $B$ ) are dominating leading ones. In the exact SMC Sudakov, the kinematic constraints guarantee that  $\Delta(Q) < 1$ . In this sense, the Sudakov in the SMC is a more exact implementation of the analytic one. We feel that the agreement apparent in Eq. (4.5) is compelling enough to proceed assuming the two Sudakov form factors are equivalent. In our phenomenological analysis, we calculate  $\Delta(Q)$  numerically.

## V. HARD EMISSION CORRECTIONS

The SMC and resummation formalisms are optimized to deal with kinematic configurations that have logarithmic enhancements  $L$ . For large  $Q_T \simeq Q$ , there are no such enhancements, and a fixed order calculation yields the most accurate predictions. The region of medium  $Q_T$ , however, is not suited to either particular expansion, in  $\alpha_s^n L^m$  or  $\alpha_s^n$ .

The problem becomes acute for SMC's. In the standard implementation of SMC's, the highest  $Q_T$  is set by the maximum virtuality allowed,  $Q = M_W$  in our example, so that the region  $Q_T \geq Q$  is never accessed. However, at  $Q_T \geq Q$ , the fixed order calculation is preferred and yields a non-zero result, so there is a discontinuity between the two predictions. Clearly, the SMC underestimates the gluon radiation well before  $Q_T \simeq Q$ , but the fixed order calculation make equally overestimate the radiation if extended to the region  $Q_T \leq Q$ .

In the  $b$ -space calculation, the same sort of behavior would occur, except that contributions to the cross section that are not logarithmically enhanced as  $Q_T \rightarrow 0$  can be added back order-by-order in  $\alpha_s$ . This procedure corrects for the approximations made in deriving the exponentiation of soft gluon emission. This correction is denoted  $Y$ , or  $Y_f$  in contrast to the resummed term  $Y_r$ . If the coefficients  $A$  and  $B$  are calculated to high-enough accuracy, one sees a relatively smooth transition between Eq. (3.1) and the NLO prediction at  $Q_T = Q$ . The situation is even better in the  $Q_T$ -space calculation, since the matching at  $Q_T = Q$  is guaranteed at any order.

It is useful to review  $Y$  in the resummed calculation, which has the form

$$Y(Q_T, Q, x_1, x_2) = \int_{x_1}^1 \frac{d\xi_1}{\xi_1} \int_{x_2}^1 \frac{d\xi_2}{\xi_2} \sum_{n=1}^{\infty} \left[ \frac{\alpha_s(Q)}{\pi} \right]^n f_a(\xi_1, Q) R_{ab}^{(n)}(Q_T, Q, \frac{x_1}{\xi_1}, \frac{x_2}{\xi_2}) f_b(\xi_2, Q). \quad (5.1)$$

The functions  $R$  at first order in  $\alpha_s$  are:

$$R_{q\bar{q}}^{(1)} = C_F \frac{(\hat{t} - Q^2)^2 + (\hat{u} - Q^2)^2}{\hat{t}\hat{u}} \delta(\hat{s} + \hat{t} + \hat{u} - Q^2) - \frac{1}{Q_T^2} \hat{P}_{q \rightarrow q}(z_B) \delta(1 - z_A) - (A \leftrightarrow B),$$

$$R_{gq}^{(1)} = \frac{1}{2} \frac{(\hat{s} + \hat{t})^2 + (\hat{t} + \hat{u})^2}{-\hat{s}\hat{u}} \delta(\hat{s} + \hat{t} + \hat{u} - Q^2) - \frac{1}{Q_T^2} \hat{P}_{g \rightarrow q}(z_B) \delta(1 - z_A),$$

and

$$R_{qg}^{(1)} = \frac{1}{2} \frac{(\hat{s} + \hat{u})^2 + (\hat{t} + \hat{u})^2}{-\hat{s}\hat{t}} \delta(\hat{s} + \hat{t} + \hat{u} - Q^2) - \frac{1}{Q_T^2} \hat{P}_{g \rightarrow q}(z_A) \delta(1 - z_B),$$

with

$$\hat{t}/Q^2 = 1 - 1/z_B \sqrt{1 + Q_T^2/Q^2}; \hat{u}/Q^2 = 1 - 1/z_A \sqrt{1 + Q_T^2/Q^2},$$

$$\hat{P}_{q \rightarrow q}(z) = C_F \left( \frac{1 + z^2}{1 - z} \right), \hat{P}_{g \rightarrow q}(z) = \frac{1}{2} [z^2 + (1 - z)^2]. \quad (5.2)$$

The first term in each expression is proportional to the squared matrix element for the hard emission, while the terms proportional to  $Q_T^2$  are the asymptotic pieces from the Sudakov form factor. Similar corrections can be derived for the SMC. In general, the hard emission contributes to the hadronic cross section proportional to

$$\frac{dx_a dx_b}{\hat{s}} f_{a/h_1}(x_a, Q) f_{b/h_2}(x_b, Q) \sigma_0 A_{exact},$$

where  $\sigma_0$  is the leading order partonic cross section, and the  $A_{exact}$  are the expressions multiplying the delta functions in Eq. (5.2). The showering contribution to the same order (assuming showering of only one parton with virtuality  $-\hat{t}$ ) is

$$dx_a d\eta_b \delta(x_a \eta_b - Q^2/S) f_{a/h_1}(x_a, t) f_{b/h_2}(\eta_b, t) \sigma_0 \frac{dz}{\hat{t}z} \hat{P}(z) \frac{f_{b/h_2}(\eta_b/z, Q)}{f_{b/h_2}(\eta_b, q)} \Delta(-\hat{t}),$$

or, by changing variables to  $x_b = \eta_b/z$ ,

$$\frac{dx_a dx_b}{\hat{s}} f_{a/h_1}(x_a, Q) f_{b/h_2}(x_b, Q) \sigma_0 A_{shower}.$$

As expressed in the functions  $R$  in Eq. (5.2), the single, hard emissions generated by the showering can be subtracted from the exact squared amplitude to include the remaining NLO corrections not present in the modified PDF. This defines a  $Y$  term for showering.

The effects of  $Y$  are included by generating the NLO subprocesses and performing the subtraction of showering contributions on an event-by-event basis. We illustrate this explicitly for the  $q\bar{q}' \rightarrow Wg$  subprocess. Each event receives an additional weight  $f_Y$  before it is accepted or rejected (this is accomplished in the subroutine `PYEVWT` already provided in the `PYTHIA` code for the user to reweight any process), where  $f_Y$  is defined as:

$$f_Y \equiv \frac{A_{exact} - A_{shower}}{A_{exact}},$$

$$A_{exact} = \frac{(\hat{t} - Q^2)^2 + (\hat{u} - Q^2)^2}{\hat{t}\hat{u}}, \quad A_{shower} = \hat{P}_{q \rightarrow q}(z) \left( \frac{\hat{s}}{-\hat{t}} \Delta(-\hat{t}) + \frac{\hat{s}}{-\hat{u}} \Delta(-\hat{u}) \right) \quad (5.3)$$

with  $z = Q^2/\hat{s}$ . The correction for  $qg \rightarrow Wq'$  has a similar form:

$$A_{exact} = \frac{(\hat{s} + \hat{t})^2 + (\hat{t} + \hat{u})^2}{-\hat{s}\hat{u}}, \quad A_{shower} = \hat{P}_{g \rightarrow q}(z) \frac{\hat{s}}{-\hat{u}} \Delta(-\hat{u}). \quad (5.4)$$

One can show that the showering corrections are either smaller than the exact squared amplitude or equal to it in the limits when  $\hat{u}$  or  $\hat{t} \rightarrow 0$ . In writing this expression, we are ignoring the possibility that later emissions are harder than the first one [23]. It has been shown previously that virtuality-ordered showering, as implemented in `PYTHIA`, yields the hardest emission first 90% of the time [27], and we ignore this technical detail.

At this point, it is useful to compare the scheme outlined above to other approaches at improving the showering algorithm. One class of corrections is based on phase-space splitting, where part of a NLO matrix element is treated with LO kinematics and part with exclusive NLO kinematics [24,25]. The idea is to allow parton showering for one of these configurations, and not the other, but particular care must be taken not to mix the different regions of phase space. There is an adjustable parameter that splits the phase space, and physical observables are sensitive to the exact choice (see the discussion in Ref. [22] regarding  $Q_T^{sep}$ ). In the approach of Ref. [24], the splitting parameter is tuned so that the contribution with LO kinematics vanishes. The resultant showering of the term with exclusive NLO kinematics generates emissions which are harder than the first “hard” emission, which is not consistent. Furthermore, the splitting parameters must be retuned for different processes and different colliders. This scheme is guaranteed to give the NLO cross section before cuts, but does not necessarily generate the correct kinematics.

The other class of corrections modifies the showering to reproduce the hard emission limit [26,27]. While this can be accomplished, it does so at the expense of transferring events from low  $Q_T$  to high  $Q_T$ . There is no attempt to predict the absolute event rate, but only to generate the correct event shapes. In some implementations, the high scale of the showering is increased to the maximum virtuality allowed by the collider energy. This is contrary to the analytic calculations, where the scale  $Q = M_W$ , for example, appears naturally (in the choice of constants  $C_1, C_2$  and  $C_3$  which eliminate potentially large logarithms). This scheme will generate the correct hard limit, but will not generate the correct cross section in the soft limit.

The present formulation contains the positive features of both schemes. Phase space is split (but without any adjustable parameters – which is also true for the corrections outlined in Ref. [23]), but higher-order corrections are also applied to the showering algorithm. This is

contained in the modified PDF. Instead of applying corrections to the showering to reproduce the hard limit, corrections are applied to the hard emission cross section to avoid double counting with the showering. The corrections allow a smooth transition between the explicit hard emission and the showering.

So far, the discussion has been theoretical. In the next section, we demonstrate the phenomenological implementation of these ideas.

## VI. NUMERICAL RESULTS

We have applied the showering modifications outlined above to  $W$  and Higgs boson production in Run I and Run II at the Tevatron, using a modified version of the `PYTHIA` Monte Carlo [3]. Most of the previous discussion applies exactly to the case of  $Z$  boson production. In particular, the modified PDF used for the showering and the corrections  $f_Y$  are exactly the same. For Higgs boson production, we use the expressions for the Wilson coefficients presented in Ref. [28]. Some technical points should be noted. First, we have not attempted to modify the Sudakov form factor implemented in `PYTHIA`; we have only modified the PDF's that drive the showering. The generalization of showering to NLO has only been accomplished for final state showering [29], and is technically complicated. Secondly, the expression for  $f_Y$ , with  $z = Q^2/\hat{s}$  and  $\Delta(Q)$  calculated numerically using Eq. (2.3), does not vanish fast enough as  $Q_T \rightarrow 0$ . This means that  $f_Y$  begins to increase sharply before vanishing at  $Q_T \simeq 0$  because of phase space. We do not believe this rise is physical, so we force the vanishing of  $f_Y$  below a certain cutoff. For all cases considered in this study, a 10 GeV cutoff is adequate. We allow  $Y$  to shower with a maximum virtuality fixed at  $-\hat{t}$  or  $-\hat{u}$ , and include primordial transverse momentum for the incoming partons, so this cut is smeared out. Ideally, however, no cut would be necessary, and this issue is left for future study. Finally, we have only considered leptonic decays of the  $W$  and the  $\gamma\gamma$  decay of the Higgs boson (to avoid any effects of final state radiation), but our final results are scaled to the total production cross section. Our results for  $Z$  boson production are similar to those for  $W$  boson production, so we do not comment on them further.

For our numerical results, we present the  $Q_T$  distribution of the heavy boson produced at the Tevatron. These distributions are in good agreement with analytic calculations, but cannot be predicted accurately by the standard showering algorithm. Secondly, we present jet properties for the same processes, which are not significantly altered from the predictions of the standard showering algorithm. These cannot be predicted by analytic calculations.

## A. Heavy Boson Properties

Our first goal is to test the predictions of our showering algorithm on Run I data. In Fig. 1, the transverse momentum of the  $W$  boson (solid line) is shown in comparison to  $D\bar{O}$  data [31] (the three dashed lines show the data and the upper and lower error estimates). The modified PDF (mPDF) is calculated using CTEQ4M PDF's. As in analytic calculations, the position of the peak from the SMC depends on non-perturbative physics [32]. In PYTHIA, this is implemented through a Gaussian smearing of the transverse momentum of the incoming partons. To generate this plot, we have changed the default Gaussian width from .44 GeV to 3.6 GeV, which is more in accord with other analyses. This is the value used in all subsequent results. The numerical agreement between our prediction and the data is very good, except, perhaps, in the region of 30–50 GeV.<sup>1</sup> Our prediction in this region depends on the details of the correction  $f_Y$ , and any excess is probably related to the bad behavior of  $f_Y$  at very small  $Q_T$ .

Next, we present our results on the  $Q_T$  distributions for  $W$  and Higgs bosons ( $m_H = 100$  GeV) produced at the Tevatron in Run II. Figure 2 displays the  $Q_T$  distribution of the  $W$  boson at Run II generated using the modified version of PYTHIA. The solid line is the full distribution, and the dashed and dotted lines show the individual contributions from the corrected showering and the corrected hard emission ( $Y$ ) piece. For reference, we show the  $Q_T$  distribution using the default version of PYTHIA and CTEQ4L PDF's. Because the maximum virtuality of the showering is set to the scale  $M_W$ , the SMC contributions are suppressed beyond  $Q_T \simeq M_W$ . Note how the  $Y$  piece fills in the intermediate  $Q_T$  region down to small  $Q_T$ , where showering gives the preferred result. The total cross section predicted by PYTHIA is (18.2, 20.5, 23.9) nb using CTEQ4L, CTEQ4M, and mPDF+ $Y$ . The total increase in rate from LO to NLO is in good agreement with Ref. [22] at  $\sqrt{S} = 2$  TeV. Our numbers for  $\sqrt{S} = 1.8$  TeV (21.2 nb using mPDF+ $Y$ ) also agree with the CDF and  $D\bar{O}$  data [30].

It is interesting that the mPDF calculation, without  $Y$ , yields nearly the same rate as using just the CTEQ4M PDF (21.6 nb vs. 20.5 nb). This is anticipated by the smallness of the virtual correction in the Wilson coefficients  $\propto \pi^2 - 8$ . However, this need not be the case for different processes or different colliders.

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<sup>1</sup>This agreement also relies on using PYTHIA v6.125 or higher, which treats the showering kinematics more correctly. This correction has an even greater impact on Higgs boson production, where, previously, the transverse momentum distribution was much broader than for  $W$  boson production.

The results for the production of a Higgs boson with mass  $m_H = 100$  GeV is shown in Fig. 3. Here, the correction to the lowest order process is quite large. The total cross section predicted by PYTHIA is (0.50, 0.48, 0.94) pb using CTEQ4L, CTEQ4M, and the mPDF+Y. The final numbers are in good agreement with Ref. [34]. We have used a primordial  $k_T$  tuned to Run I data. However, if we believe that the non-perturbative function should have the same form as the perturbative Sudakov form factor, then the primordial  $k_T$  should scale like  $C_A/C_F$  relative to the  $q\bar{q}$  case [35], and the peak of the  $Q_T$  distribution would shift to the right.

It is interesting to know if the kinematic distributions for the heavy bosons can be reproduced using the standard showering algorithms with a multiplicative  $K$ -factor that yields the total NLO rate. Figure 4 shows the ratio of the Higgs boson transverse momentum distributions calculated from mPDF+Y to CTEQ4L times  $K$  ( $K \gtrsim 2$ ). There are variations as large as 10% in the important regions of small and medium  $Q_T$ . Of course, the effect is much larger for the large  $Q_T$  region where there is almost no rate from the standard parton showering.

## B. Jet properties

In Figs. 5, 6 and 7, we present jet properties for the  $W$  and Higgs boson production processes. Jets are defined using the cone clustering algorithm of the PYTHIA subroutine PYCELL with a cone of size  $R = 0.5$ ,  $E_T > 5$  GeV, and  $|\eta| < 2.5$ . For  $W$  boson production in Fig. 5, we present the jet multiplicity distributions for all  $Q_T$ ,  $Q_T < 10$  GeV,  $10 < Q_T < 20$  GeV, and  $Q_T < 30$  GeV. We compare the showering predictions using mPDF (solid line), CTEQ4M (dashed line), and CTEQ4L (dotted line). To study the effects of the modified showering algorithm, we do not include  $Y$ , which would increase the  $N_{jet} = 1$  bin of the mPDF prediction. All distributions are normalized to unity. From Fig. 5, we see that the predictions have only minor differences, which is expected since the Wilson coefficients for  $W$  production are nearly unity.

For the case of Higgs boson production in Fig. 6, we study the regions  $Q_T < 25$  GeV,  $25 < Q_T < 50$  GeV, and  $Q_T < 75$  GeV. There are more noticeable differences, and much more radiation in general than for the  $W$  boson case. The higher order PDF's generate slightly more jet activity and yield similar distributions. In general, though, there are no dramatic changes in the distributions. This is not too surprising, since the showering depends on the ratio of the modified PDF's evaluated at two different scales, which is not as sensitive to the overall normalization of the PDF.

From these examples, we learn that the use of the modified showering algorithm does not change the jet properties in a major way. Figure 7 compares the differential jet transverse energy distribution in Run I to CDF data. Here, jets are defined with  $R = 0.4$  and are smeared with an energy resolution function  $\Delta E_T/E_T = 1.2/\sqrt{E_T}$  (in GeV). We consider the agreement between theory and data to be further evidence that our NLO corrections reproduce jet properties.

## VII. CONCLUSIONS

We have presented a modified, parton showering algorithm that produces the total cross section and the event shapes beyond the leading order. These modifications are based on the  $Q_T$ -space resummation. The parton showering itself is modified by using a new PDF (called mPDF) which encodes some information about the hard scattering process. Simultaneously, the explicit, hard emission is included, but only after subtracting out the contribution already generated by the showering: this correction is called  $Y$ . The presence of  $Y$  yields a smooth transition from the parton showering to single, hard emission. We modified the PYTHIA Monte Carlo to account for these corrections, and presented comparisons with Run I  $W$  boson data and predictions for  $W$  and Higgs boson production at the Tevatron in Run II.

The scheme works very well at NLO for the cases considered in this study, and the correct cross sections, transverse momentum distributions, and jet properties are generated. We have compared our kinematic distributions to the case when the results of the standard showering are multiplied by a constant  $K$ -factor to reproduce the NLO cross section. We find variations on the order of 10% for small and medium transverse momentum.

There are several effects which still need study. We have not included the exact distributions for the decay of the leptons [36] for  $W$  and  $Z$  production, which are resummed differently. It is straightforward to include such effects. In the theoretical discussion and numerical results, we have focussed on initial state radiation, but our results should apply equally well for final state radiation. The situation is certainly simpler, since final state radiation does not require detailed knowledge of the fragmentation functions. Also, the case when color flows from the initial state to the final state requires study. A resummed calculation already exists for the case of deep inelastic scattering [37], and much theoretical progress has been made for heavy quark production [38]. We believe that the modified showering scheme outlined in this study generalizes beyond NLO, just as the analytic calculations can be calculated to any given order. For example, we could include hard  $W + 2$  jet corrections

[39] to  $Y$ . For consistency, however, higher order terms ( $A$  and  $B$ ) may also need to be included in the Sudakov form factor.

The modified `PYTHIA` subroutines used in this study and an explanation of how to use them are available at the web address [moose.ucdavis.edu/mrenna/shower.html](http://moose.ucdavis.edu/mrenna/shower.html).

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# FIGURES

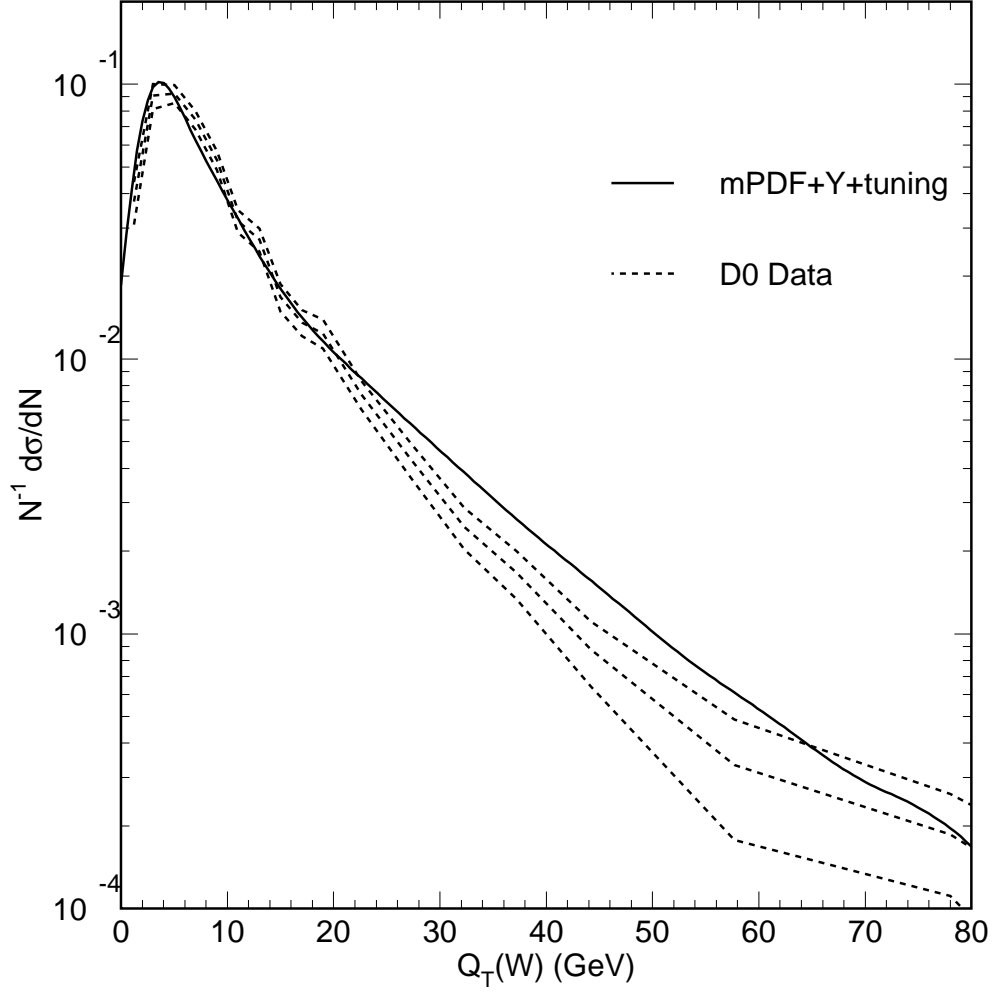


FIG. 1. The prediction of the  $W$  boson transverse momentum distribution in Run I at the Tevatron (solid line) compared to the DØ data (the upper and lower dashed lines represent the errors on the middle line). The prediction includes the effects of the modified parton distribution functions, the correction to the hard scattering process, and a tuned primordial  $k_T$  of 3.6 GeV.

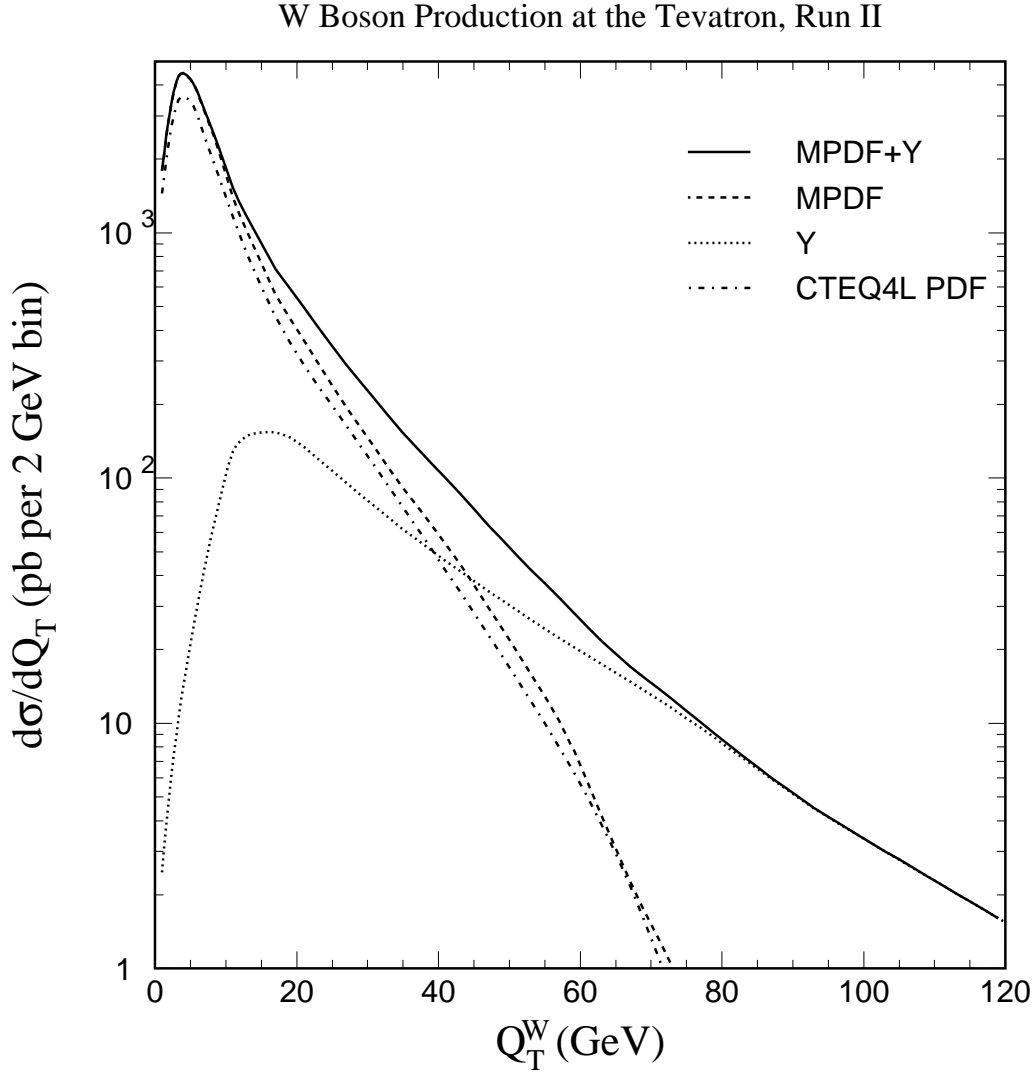


FIG. 2. Predicted  $W$  boson transverse momentum distribution in Run II (solid line) showing the individual contributions from showering (long dashes), the corrected hard emission (short dashes) and the standard PYTHIA prediction using CTEQ4L structure functions (dot-dash).

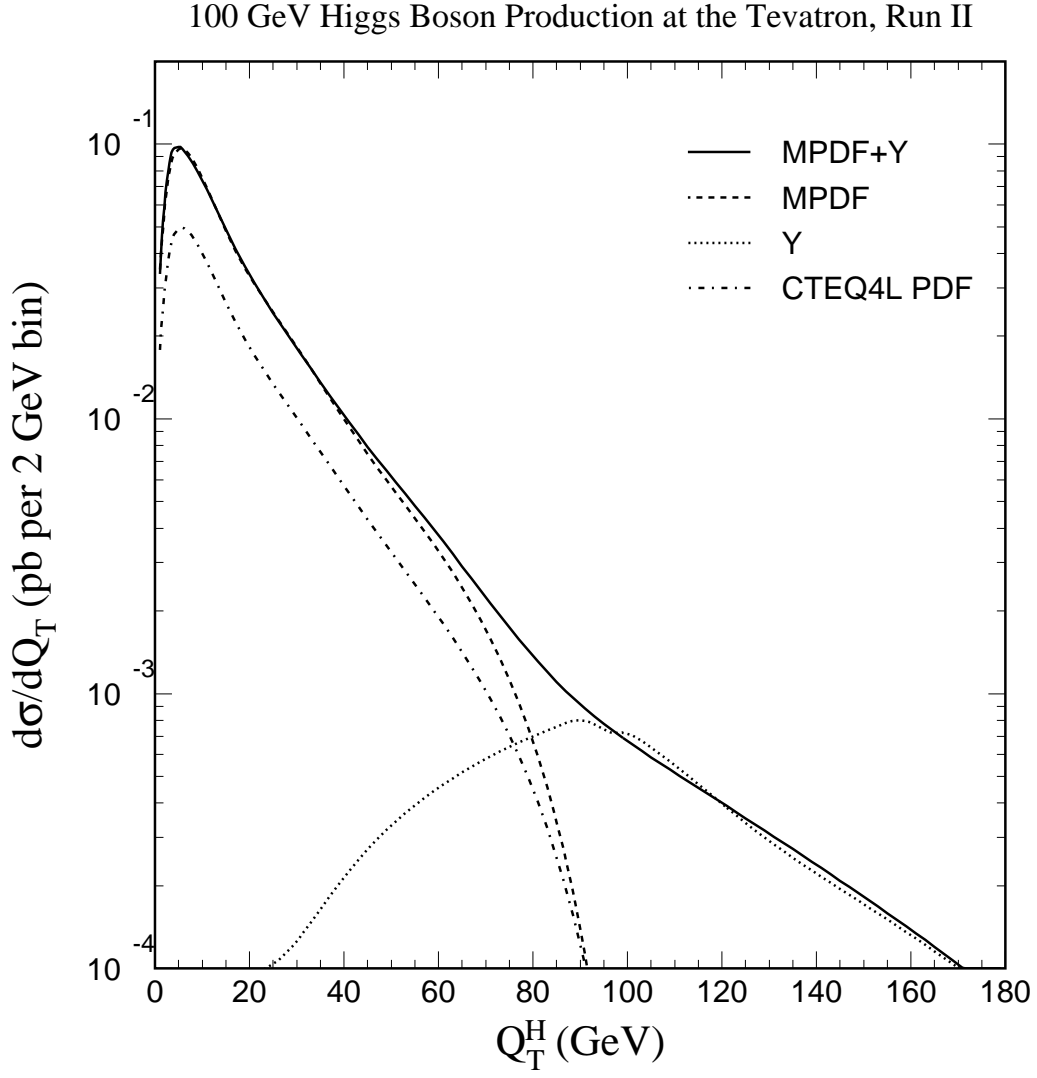


FIG. 3. Same as Fig. 2, but for Higgs boson production ( $m_H = 100$  GeV).

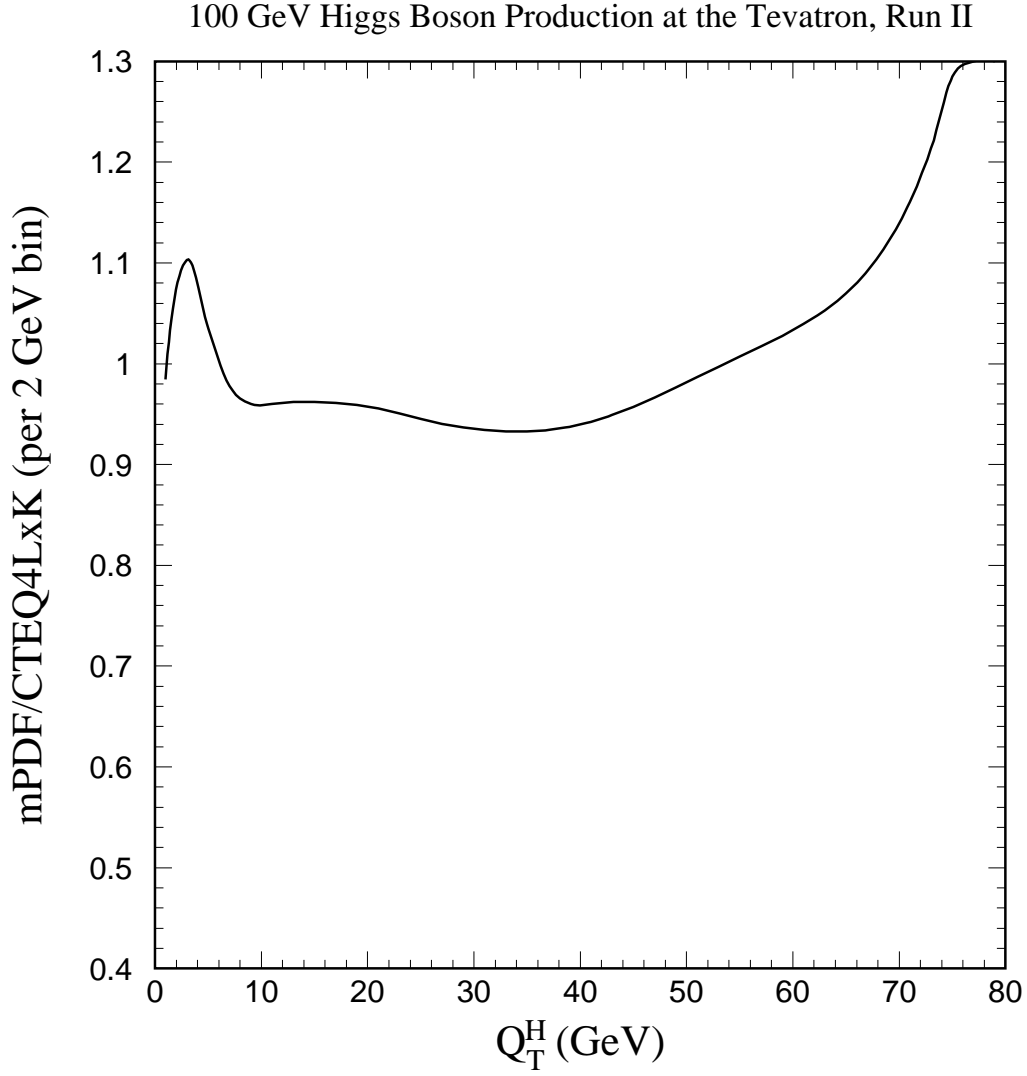


FIG. 4. The ratio of Higgs boson transverse momentum distributions ( $m_H = 100$  GeV) from the modified showering algorithm and from the standard showering algorithm using the CTEQ4L PDF. The CTEQ4L result has been multiplied by a constant  $K$ -factor to reproduce the NLO rate.

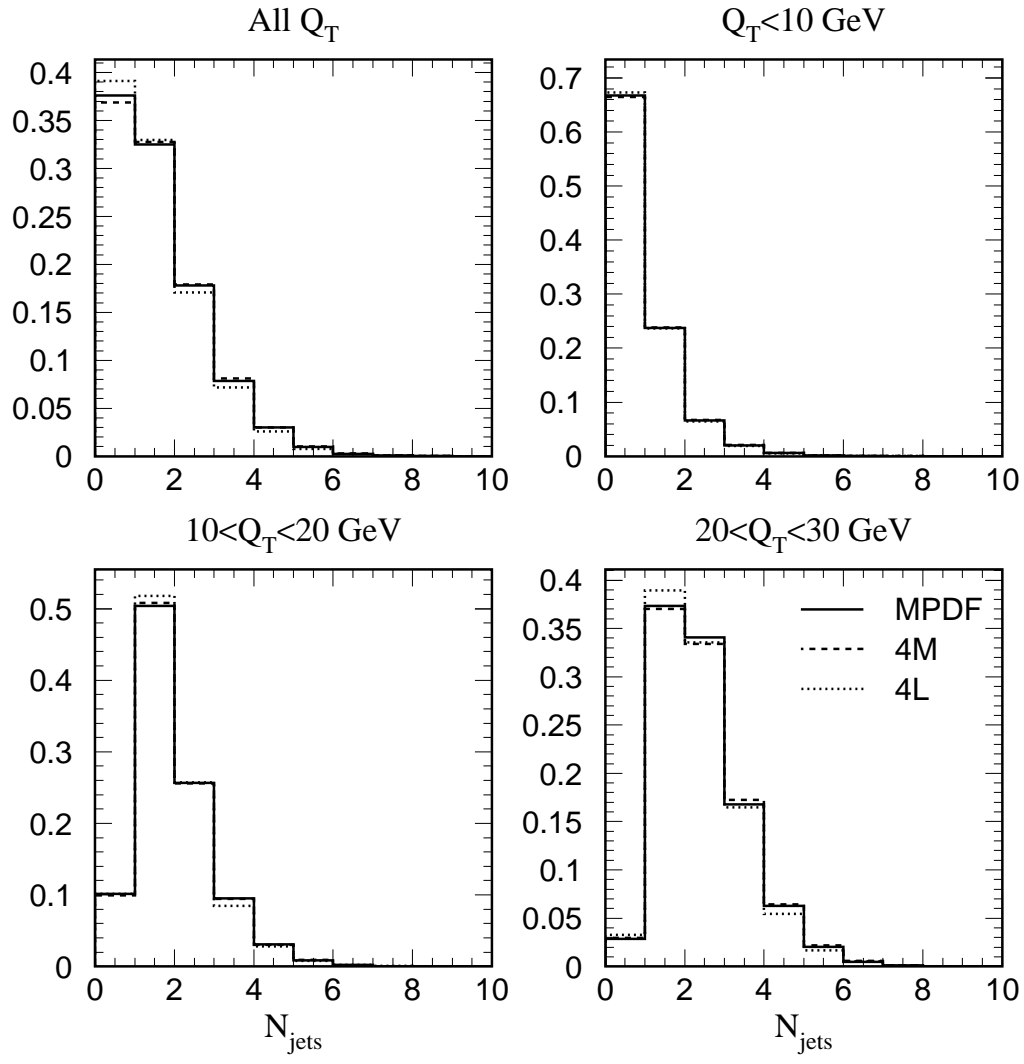


FIG. 5. Jet multiplicity in  $W$  boson events for different  $Q_T$  binnings.



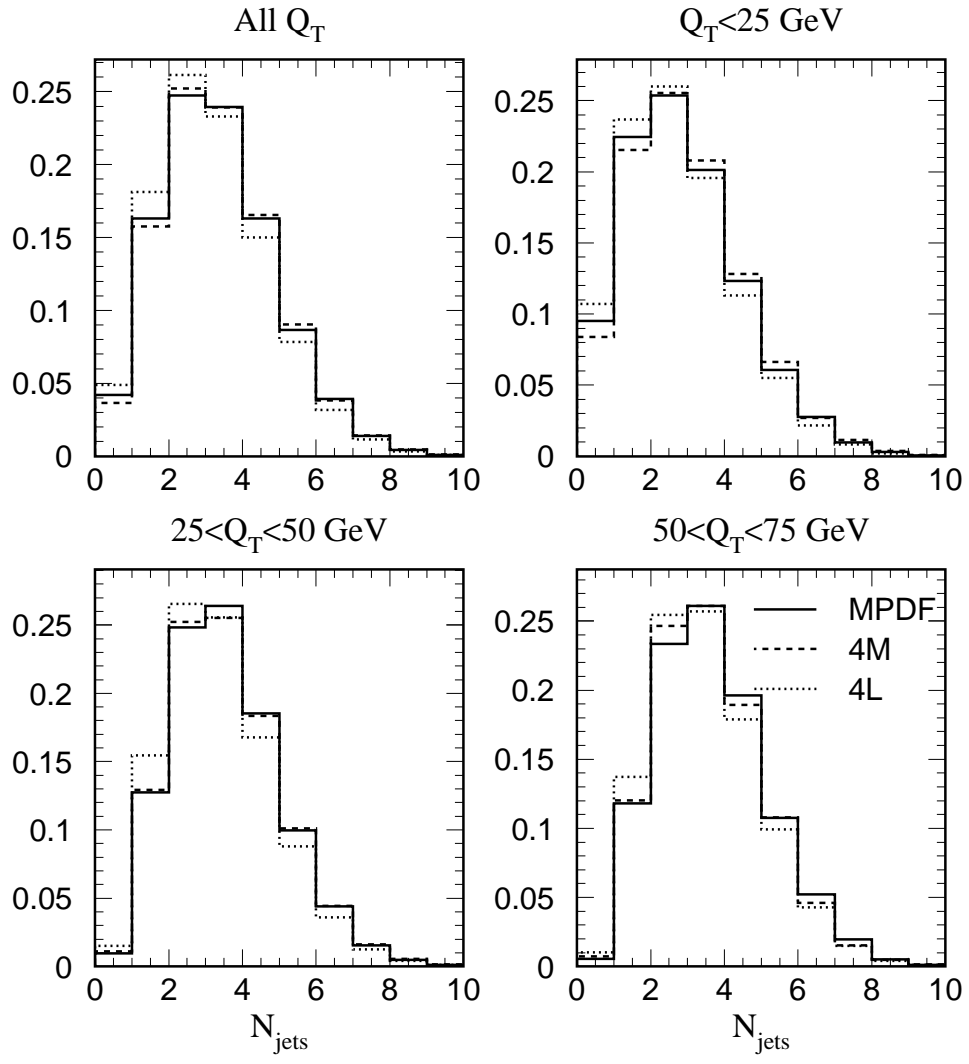


FIG. 6. Jet multiplicity in Higgs boson events for different  $Q_T$  binnings.

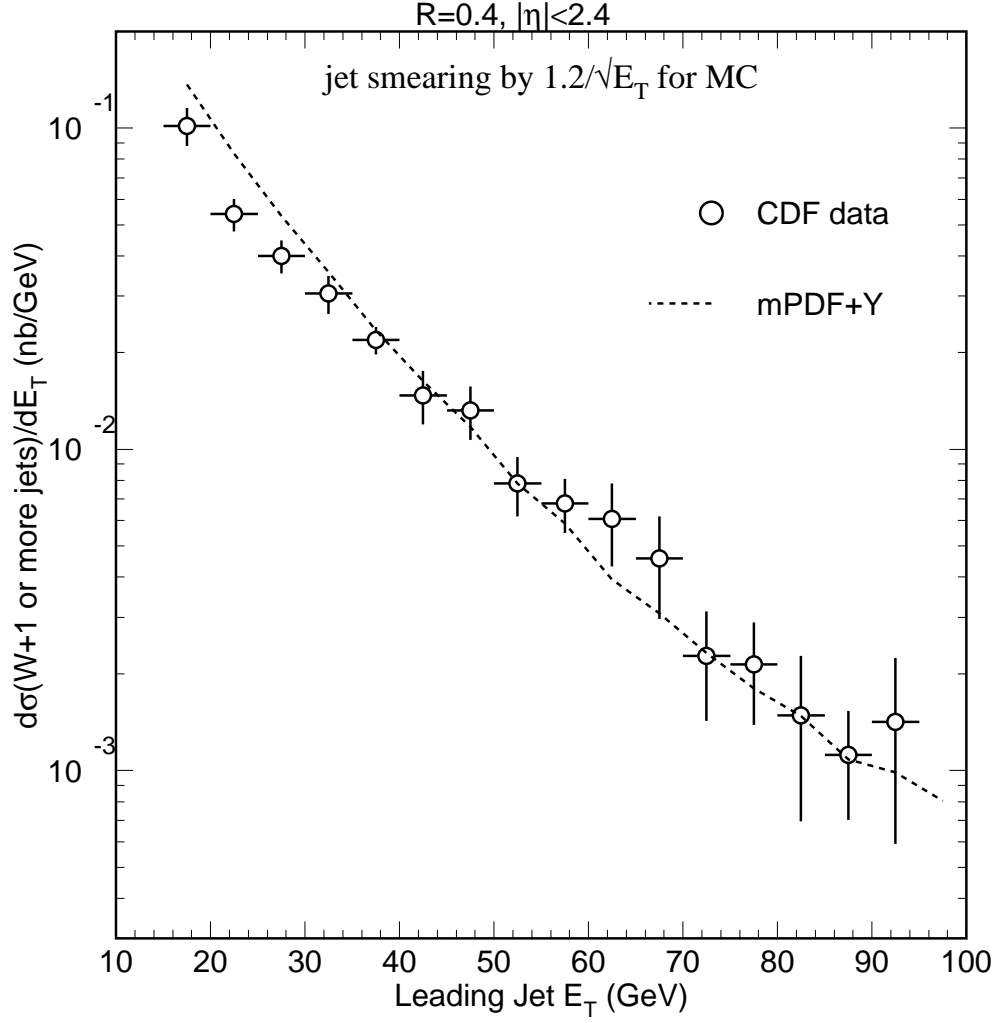


FIG. 7. Comparison of the differential jet cross section in  $W$  boson + jet events in Run I (dashed line) to the CDF data (circles). The jets have been smeared by the resolution  $\Delta E_T/E_T = 1.2/\sqrt{E_T}$  (in GeV).